**Barron’s Let’s Review Regents – Algebra II**

# Chapter 7: Functions

## 7.1 Composite Functions

**Key Ideas**

A function is a mathematical rule that generally takes a number input and then outputs a number. If the function is called f, the notation means that when the number 4 is put into the function, the number 11 is output from the function. Functions can be used to describe mathematical relationships, including real-world scenarios.

Functions are often defined by a formula. For example, the formula means that when a number is put into the function, a number that is three more than twice that number will come out of the function. So .

Not only can numbers be put into a function but so can variables or even other functions. Using the function defined above, and .

If another function is defined as , it is possible to create a new function . By putting into the function, the new function becomes .

When a function is put into another function, the result is called a composite function. When working with composite functions, start with the inner function first and then move to the outer function.

**Example 1**

If and , what is the value of ?

*Solution*:

Since .

**Example 2**

If and , what are and ?

*Solution*:

Notice that in this example, is not equivalent to .

**Example 3**

If and , what is ?

Solution:

More difficult than finding the composition of two given functions is trying to find two functions whose composition would become some given function. For example, could be *decomposed* into where and . There are other ways to decompose this function into two functions also, but this way is the most useful.

**Example 4**

Which of the following could be and if

(1)

(2)

(3)

(4)

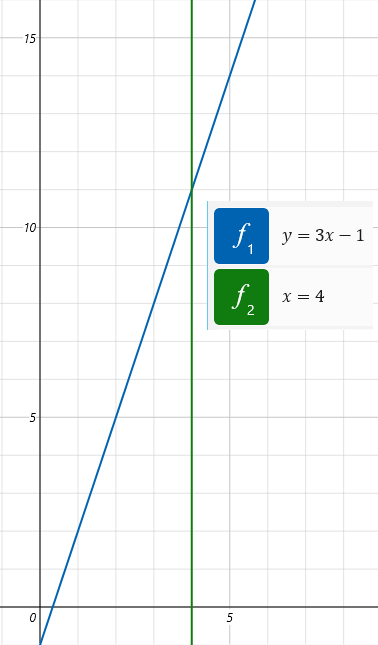
*Solution*: Choice (2) is correct since   
. If you test choice (1), it would be come .

**Composite Functions on the Graphing Calculator**

Graphing calculators can also evaluate functions and composite functions.

Evaluate the function at .

**Using Microsoft Windows Calculator and Microsoft Paint**



**Using Geogebra**

A graph of a line

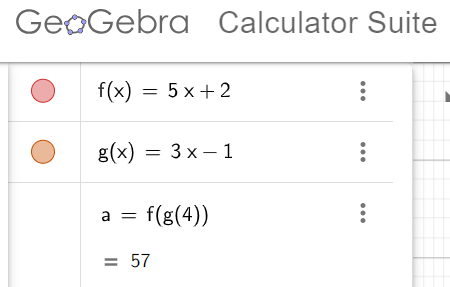
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A screenshot of a calculator suit

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To evaluate composite functions like when and , both functions need to be entered into the calculator.

**Using Geogebra**



**Odd and Even Functions**

A function is called an *even* function if its graph has -axis symmetry. A function is called an odd function if its graph has *origin* symmetry. The graphs of most functions have neither of these symmetries and so are neither odd nor even.

**Average Rate of Change**

A function has something called the average rate of change on an interval. This is very similar to the concept of slope discussed in Chapter 1.

The formula for the average rate of change of the function on the interval to is shown:

**Example 5**

What is the average rate of change of the function on the interval ?

*Solution*: Use the formula:

**End Behavior of Functions**

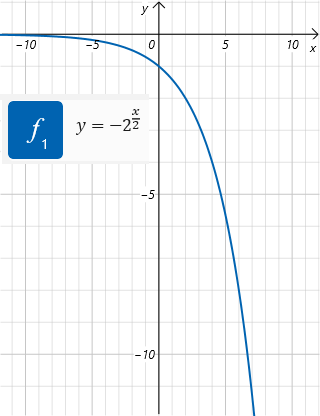
The end behavior of a function is a description of what output values happen when very large positive or very negative values are put into the function. When a large number is put into the function   
, for example , the function outputs a very large positive number.

Using symbols, we write: As , where the means infinity.

When a large negative number is put into the same function, like , the function outputs a very large positive number.

Using symbols, we write: As .

The end behavior can be seen on the graph of the function.



For large positive input values, this function outputs a large negative number. For large negative input values, this function outputs a number close to zero.

The end behavior of this function can be described:

As   
As

### Check Your Understanding of Section 7.1

1. *Multiple-Choice*
2. If and , what is ?  
   **(3) 25**
3. If and , what is ?  
   **(2) 7**
4. Which of the following could be and if **(1)**
5. If , what is ?  
   **(4) Not enough information to answer**
6. If , what is ?  
   **(4)**
7. If , what is the value of?  
   **(1) 126**
8. If , what is ?  
   **(1)**
9. If , what is g(f(x))?  
   **(3)**
10. If and , what is g(x)?  
    **(4)**
11. If , what is the value of ?  
    **(3) 26**
12. *Show how you arrived at your answers*.
13. and . Griffin says . Ruben says . Who is correct?  
    **Griffin is correct.  
    Book answer is partially incorrect. Book says Griffin is correct. 17 is not correct.**
14. If , and , what is ?
15. Given the two graphs of and , what is the value of ?  
      
    A graph of a line with points

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    A graph of a function

    AI-generated content may be incorrect.  
       
       
    Points: (0, -3), (2, 1)  
    -intercept: (0, -3)   
       
       
      
     parabola  
       
       
       
    Points: (0, 1), (1, 2), (-1, 2)  
       
       
      
       
    **I disagree with the book’s answer. . You can see it on the graph that .**A screenshot of a calculator

    AI-generated content may be incorrect.
16. If and the graph of is shown below, what does the graph of look like?  
      
    A graph of a function

    AI-generated content may be incorrect.

|  |  |  |
| --- | --- | --- |
| **x** | **g(x)** | **f(g(x))** |
| 0 | 0 | 2 |
| 2 | 5 | 7 |
| 7 | 5 | 7 |
| 9 | 0 | 2 |
| 11 | 8 | 10 |

A graph of a function

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1. If and , for what value(s) of does ?

## 7.2 Inverse Functions

**Key Ideas**

If a function has an inverse function, the inverse function can take what was output from the original function and turn it back into the original input. An inverse function is a function that undoes what some other function has done to a number.

If the function is defined by , then . When the number 10 is put into the inverse function, the output is 7. This value was put into the original function f, to get the number 10: . Whatever the function does to a number, undoes to that number.

**Example 1**

If , what is the value of ?

(1) 6

(2)

(3) 19

(4) -19

*Solution*: Choice (1) is correct. Since the function turned the number 6 into the number 19, the inverse will turn the 19 back into the 6. With the given information, this is the only value for that can be determined.

An exponent of -1 has a different meaning when it is used in functions that when it is used with numbers. Although is not . Instead, represents the inverse of function .

**Some Basic Inverse Functions**

If a function has one operation happening to the input value, the inverse will have the opposite operation happening to its input variable. Some examples of basic inverse functions are shown in the table.

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
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|  |  |
|  |  |

For any of these function/inverse pairs, you can verify that the composition

**Determining the Inverse Function of a Linear Equation**

A linear function of the form , such as , has an inverse function that is also a linear function. The process for finding the inverse function is to first rewrite the function but with the replaced with an and the replaced with an .

Then solve for . In this case, subtract 3 from both sides of the equation an divide both sides of the equation by 2.

If you pick a number for like 5, you can see that , and that   
. So, this inverse function undoes what the function did to the number 5.

**Example 2**

What is the inverse of the function ?

(1)

(2)

(3)

(4)

*Solution*: Choice (3) is correct. Replace the with and replace the with an . Solve the equation for by adding 5 to both sides of the equation and then dividing both sides of the equation by 3.

**Answering Questions About the Inverse Without Determining the Inverse**

A trick question that appears on many Regents exams is one like the following.

If , what is ?

The long way to do this question is to use the inverse process to find that the inverse function is   
 and then calculate .

Since is asking what input value for will output the number 28, a shorter way to do this is to simply solve the equation .

**Example 3**

If , what is the value of ?

Solution: Solve the equation .

**Graphs of Inverse Functions**

If, for some function then the point (3, 9) will be on the graph for that function. If , then . So, the point (9, 3) will be on the graph for the inverse function. In general, if is a point on the graph of the original function, then will be a point on the graph of the inverse function. In terms of transformations, the point (9, 3) is the reflection over the line of the point (3, 9). When this transformation is done to every point on the graph of the function, the graph of the inverse will be a reflection of the entire graph of the original function over the line .

A graph of a function

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A graph of a line graph

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Some more graphs of functions and their inverses are shown in the following table.

A graph of a function

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A graph of function and equations

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A graph of a function

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**Example 4**

Which choice is the graph of the inverse of the function whose graph is below?

A graph with a line going up

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A graph of function with lines and numbers

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Solution: Choice (4) is the answer. The graph of this choice looks like the original graph reflected over line .

**Inverse Functions on the Graphing Calculator**

**More Complicated Inverses**

Although they were not called inverse functions when introduced, two inverse functions were already presented in this book.

An exponential equation like has an inverse function. Since , the inverse function must make . The inverse of is , and does equal 3.

A graph of function and a line

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The trig function , has the inverse function . So and ,

A graph of a function

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### Check Your Understanding of Section 7.2

1. Multiple-Choice
2. If is the inverse of , which is the following is *not* true?  
   **(2)**
3. If is the inverse of , what is the value of ?  
   **(4) 9**
4. What is the inverse of ?  
   **(1)**
5. What is the inverse of ?  
   **(1)**
6. What is the inverse of ?  
   **(3)**
7. If , what must also be true?  
   **(1)**
8. If , what is ?  
   Check:   
   **(2)**
9. If the graph of is shown below, what is the graph of ?  
   Points for: :   
   (-2, -5) to (1, 4), (1,4) to (7, 6)  
   Points for:   
   (-5, -2) to (4, 1), (4,1) to (6, 7)  
   A graph of a function

   AI-generated content may be incorrect.

**(3)**A graph of a function

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1. What I the inverse of ?  
   **(4)**
2. What is the inverse of   
   ?  
   Quadrants I and IV.  
   sin is negative in Quadrant IV, and positive in Quadrant I.  
   **(3)**
3. *Show how you arrived at your answers*.
4. If , what is ?  
   Point: (6, 34)
5. If the graph of is shown below, sketch on the same axes the graph of   
   .  
   A graph of a function

   AI-generated content may be incorrect.  
    Line Segments:   
   (-5, -4) to (0, 5), (0, 5) to (8, 6)  
    Line Segments:  
   (-4, -5) to (5, 0), (5, 0) to (6, 8)  
     
   A graph of a function

   AI-generated content may be incorrect.
6. If , what is ?

Check:

1. If , what is ?  
   **The inverse function can take what was output from the original equation and turn it back into the original input.**
2. If and , show how you would solve by using the inverse functions and .  
     
   I don’t understand the logic here, and the answer in the book isn’t helping.